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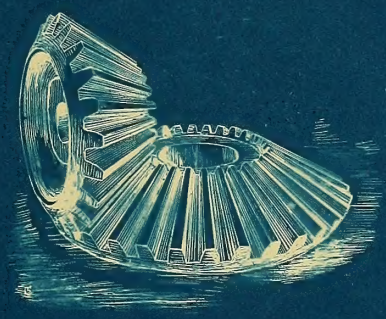
GEARING,

A

COLLECTION OF FORMULAE & SUGGESTIONS



FOR THE

DRAWING BOARD

by

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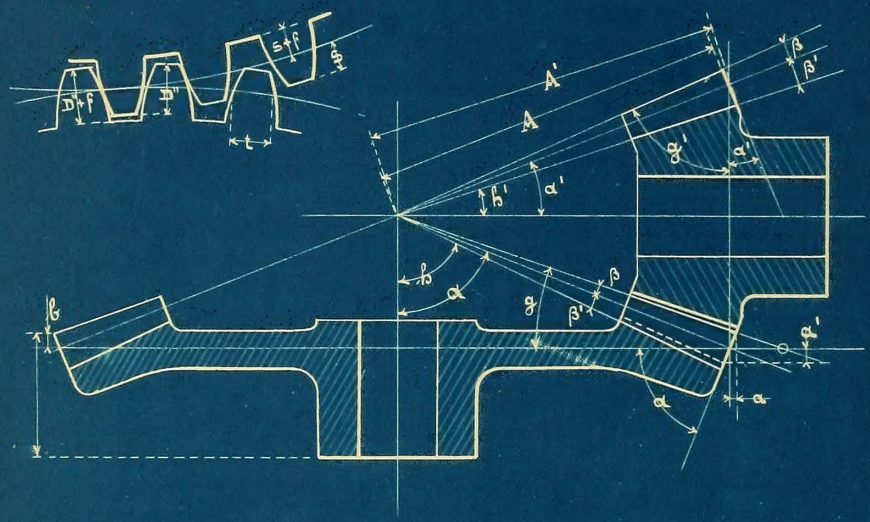


2a	Diameter increment
b	Distance from top of tooth to Pitch Line
a	Center Angle = Angle of Edge = Pitch Angle.
β	Angle of Top
β'	Angle of Bottom
g	Angle of Face
b	Cutting Angle
A	Apex-Distance on Pitch-Line
A'	Apex-distance at Bottom
P	Diametral Pitch
P'	Circular Pitch
N	Number of teeth
D"	Working depth of tooth
D"+f	Total depth of tooth
s	Addendum
f	Clearance at bottom of tooth
t	Thickness of tooth at Pitch Line
F	Width of Face
d	Pitch-diameter
d'	Outside-diameter

C.

$$2a = 2s \cos \alpha$$

$$b = \frac{a}{\sin \alpha} \qquad \begin{cases} a \text{ for gear} = b \text{ for pinion} \\ a \text{ for pinion} = b \text{ for gear} \end{cases}$$

$$\operatorname{tg} \alpha = \frac{N \text{ in Gear}}{N \text{ in Pinion}} \quad ; \quad \operatorname{tg} \alpha' = \frac{N \text{ in Pinion}}{N \text{ in Gear}}$$

$$\operatorname{tg} \beta = \frac{s}{A} \qquad \operatorname{tg} \beta' = \frac{s+f}{A}$$

$$\gamma = 90^\circ - (\alpha + \beta) = \alpha' - \beta$$

$$A = \sqrt{(\text{Rad. of Pinion})^2 + (\text{Rad. of Gear})^2}$$

$$A' = \sqrt{A^2 + (s+f)^2} = \frac{A}{\cos \beta'}$$

$$A = \frac{N}{2P \sin \alpha}$$

$$A' = \frac{N}{2P \sin \alpha \cos \beta'}$$

$$s = A \operatorname{tg} \beta$$

$$\operatorname{tg} \beta = \frac{2 \sin \alpha}{N}$$

$$A = \frac{\frac{1}{2} d'}{\sin (\alpha + \beta)} \cos \beta$$

$$P = \frac{N}{2A \sin \alpha}$$

$$P' = \frac{\pi}{P} \qquad P = \frac{\pi}{P'} \qquad D'' = 2s$$

$$s = \frac{P'}{\pi} = .3183 P' = \frac{1}{P}$$

$$s + f = A \operatorname{tg} \beta' \qquad s + f = .3685 P'$$

$$t = \frac{P'}{2} = \frac{\pi}{2P}$$

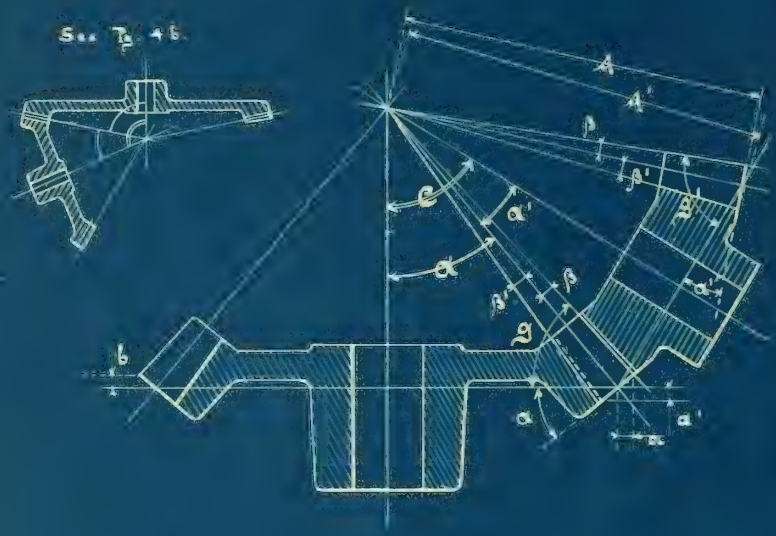
$$F = \frac{4}{P} + \frac{A}{7} \qquad F = 2P' \text{ to } 3P'$$

$$f = \frac{t}{10}$$

Note: For planed gears calculate s and $s+f$

C.

See Pg. 46.



2a	Diameter increment
b	Distance from top of tooth to P. line
c	Angle formed by axis of gears
a, a'	Angle of Edge, Center - or Pitch - angle } gear, pinion
β	Angle of Top
β'	Angle of Bottom
g	Angle of Face
A	Apex - distance at Pitch - line
A'	Apex - distance at Bottom
P	Diametral Pitch
P'	Circular Pitch
N	Number of teeth in gear
n	Number of teeth in pinion
D"	Working - Depth of tooth
D"+f	Total Depth of tooth
s	Addendum
f	Clearance of tooth at bottom
t	Thickness of tooth at Pitch - line
d	Pitch - Diameter

$$s = \frac{P'}{\pi} = .3183\,P' = \frac{1}{P}$$

$$d = \frac{N}{P}$$

$$A = \frac{N}{2P\sin\alpha} = \frac{n}{2P\sin\alpha'}$$

$$\operatorname{tg}\alpha = \frac{\sin c}{\frac{n}{N} + \cos c} \quad \text{or} \quad \operatorname{ctg} d = \frac{n}{N\sin c} + \operatorname{ctg} c$$

$$\operatorname{tg}\alpha' = \frac{\sin c}{\frac{N}{n} + \cos c} \quad \text{or} \quad \operatorname{ctg} d' = \frac{N}{n\sin c} + \operatorname{ctg} c$$

$$\operatorname{tg}\beta = \frac{s}{A}$$

$$\operatorname{tg}\beta = \frac{2\sin\alpha}{N} = \frac{2\sin\alpha'}{n}$$

$$\operatorname{tg}\beta' = \frac{s+f}{A}$$

$$\operatorname{tg}\beta' = \frac{\sin\alpha(20+\pi)}{10\,N} = \frac{2.314\sin\alpha}{N}$$

$$t = \frac{P'}{2} = \frac{\pi}{2P}$$

$$f = \frac{t}{10}$$

$$s+f = \frac{1}{P}\left(1+\frac{\pi}{20}\right)$$

$$A' = \frac{A}{\cos\beta'}$$

$a = s\cos\alpha$ for gear

$a = s\cos\alpha'$ " pinion

$$b = \begin{cases} a \text{ for gear} = b \text{ for pinion} \\ a \text{ " pinion} = b \text{ " gear} \end{cases}$$

$$g = 90 - (\alpha + \beta)$$

c

Note: Calculate s + $s+f$.

The formulae given for α & α' (when c, N & n known) undergo some modifications for values of c greater 90°.

For bevel-gears at any angle but 90° we may distinguish 4 cases; c, N & n given:

<u>I. Case</u>. See page 4a.

<u>II. Case</u>. When c greater 90°

$$tg\,\alpha = \frac{\sin(180-c)}{\frac{n}{N} - \cos(180-c)}\;;\quad tg\,\alpha' = \frac{\sin(180-c)}{\frac{N}{n} - \cos(180-c)}$$

<u>III. Case</u>. $\alpha = 90°$; $\alpha' = c - 90°$

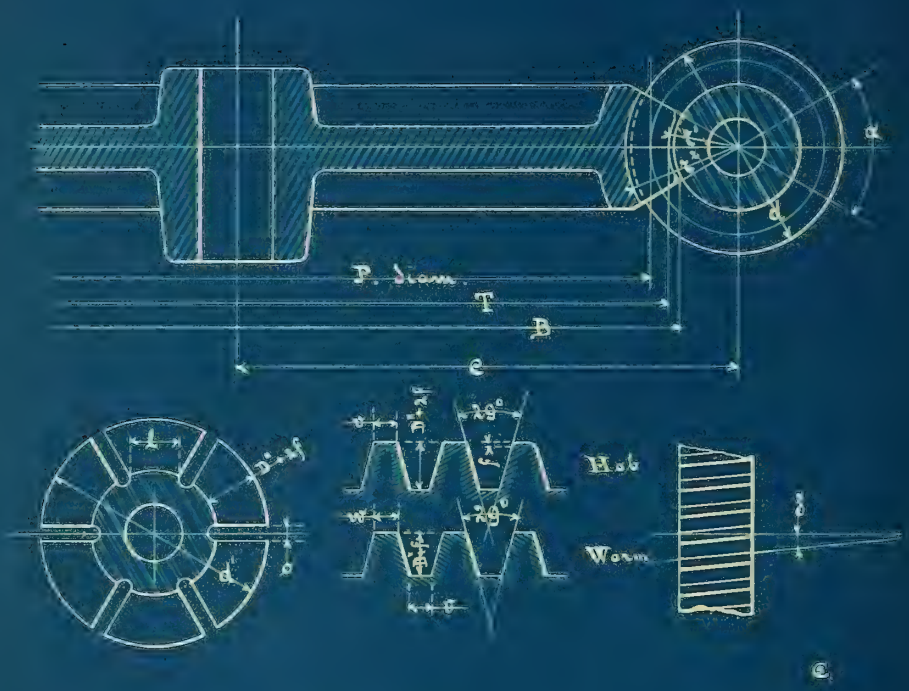
<u>IV. Case</u>.

$$tg\,\alpha = \frac{\sin E}{\cos E - \frac{n}{N}}\;;\quad tg\,\alpha' = \frac{\sin E}{\frac{N}{n} - \cos E}$$

For an example to belong to case 3, the following condition must be fulfilled:

$$N \sin(c - 90) = n$$

If $N \sin(c-90)$ is <u>smaller</u> than <u>n</u>, we have case <u>II.</u>
" " " <u>larger</u> " <u>n</u> " " " <u>IV.</u>



L	Lead of Worm
N	Number of teeth in gear
m	Threads per inch in worm
d	Diameter of worm
d'	Diameter of hob
T	Throat - Diameter
B	Blank - Diameter (to sharp corner)
C	Distance between Centers
s	Thickness of hob - slotting Cutter
l	Width of lands at bottom
b	Pitch - Circumference of Worm
v	Width of Worm - thread tool at end
w	Width of Worm - thread at top
P	Diametral Pitch
P'	Circular Pitch

If Lead is for single, double, triple etc. thread, then:

$$L = P', \; 2P', \; 3P' \text{ etc.}$$

$$a = 60^\circ \text{ to } 90^\circ$$

$$r' = \frac{d}{2} - 2s$$

$$s = \frac{1}{P} = \frac{P'}{\pi} = .3183P$$

$$P = \frac{\pi}{P'}$$

$$r^2 = r' + D'' + f$$

$$\frac{NP'}{\pi} = \frac{N}{P} = \text{Pitch-Diameter}$$

$$P' = \frac{\pi T}{N+2}$$

$$b = \pi(d - 2s)$$

$$\operatorname{tg}\delta = \frac{L}{b}$$ Practical only when width of wheel or Wheel-P.-circle is not more than 2/3 P.-Diam. of Worm.

$$T = \frac{N}{P} + 2s$$

$$B = T + 2\left(r' - r'\cos\frac{\alpha}{2}\right)$$ A measurement of sketch is often sufficient.

$$C = \frac{P.\text{-Diam}}{2} + r' + s$$

$$o = \frac{.335\,P'}{2} + \frac{1}{8}''$$

$$l = D'' + 2f + \frac{1}{8}''$$

$$v = .31\,P'$$

$$w = .335\,P' \qquad L = \frac{1}{m}$$

$$d' = d + 2f$$

Note: Nut + Worm should be marked: (Example)

4 Thrds. per 1" single .25 P., .25 L.
2 " " 1" double .25 P., .50 L.

TABLE OF TOOTH PARTS FOR GEARS AND CUTTERS.

DIAMETRAL PITCH P

P	t	D"+f	D"	s	s+f
1/4	6.2832	8.6283	8.0000	4.0000	4.6283
1/2	3.1416	4.3142	4.0000	2.0000	2.3142
3/4	2.0944	2.8761	2.6666	1.3333	1.5428
1	1.5708	2.1571	2.0000	1.0000	1.1571
1 1/4	1.2566	1.7257	1.6000	.8000	.9257
1 1/2	1.0472	1.4381	1.3333	.6666	.7714
1 3/4	.8976	1.2326	1.1429	.5714	.6612
2	.7854	1.0785	1.0000	.5000	.5785
2 1/4	.6981	.9597	.8888	.4444	.5143
2 1/2	.6283	.8628	.8000	.4000	.4628
2 3/4	.5712	.7844	.7273	.3636	.4208
3	.5236	.7190	.6666	.3333	.3857
3 1/2	.4488	.6163	.5714	.2857	.3306
4	.3927	.5393	.5000	.2500	.2893
5	.3142	.4314	.4000	.2000	.2314
6	.2618	.3595	.3333	.1666	.1928
7	.2244	.3081	.2857	.1429	.1653
8	.1963	.2696	.2500	.1250	.1446
9	.1745	.2397	.2222	.1111	.1286
10	.1571	.2157	.2000	.1000	.1157
11	.1428	.1961	.1818	.0909	.1052
12	.1309	.1798	.1666	.0833	.0964
13	.1208	.1659	.1538	.0769	.0890
14	.1122	.1541	.1429	.0714	.0826
15	.1047	.1438	.1333	.0666	.0771
16	.0982	.1348	.1250	.0625	.0723
17	.0924	.1269	.1176	.0588	.0681
18	.0873	.1198	.1111	.0555	.0643
19	.0827	.1135	.1053	.0526	.0609
20	.0785	.1079	.1000	.0500	.0579
21	.0748	.1026	.0952	.0476	.0550
22	.0714	.0980	.0909	.0454	.0526
23	.0683	.0936	.0868	.0434	.0502
24	.0654	.0898	.0833	.0417	.0482
25	.0628	.0863	.0800	.0400	.0463
26	.0604	.0829	.0769	.0385	.0445
27	.0581	.0798	.0740	.0370	.0428
28	.0561	.0770	.0714	.0357	.0413
29	.0541	.0742	.0688	.0344	.0398
30	.0524	.0719	.0666	.0333	.0386
31	.0506	.0695	.0644	.0322	.0372
32	.0491	.0674	.0625	.0312	.0362
33	.0476	.0653	.0606	.0303	.0350
34	.0462	.0634	.0588	.0294	.0340
35	.0448	.0615	.0570	.0285	.0329
36	.0436	.0599	.0555	.0278	.0321
37	.0424	.0582	.0540	.0270	.0312
38	.0413	.0568	.0526	.0263	.0304
39	.0402	.0552	.0512	.0256	.0296
40	.0393	.0539	.0500	.0250	.0289

CIRCULAR PITCH P'

THREADS PER IN.	P'	P'	P	t	D"+f	s	s+f
1/2	2.000	2	1.571	1.000	1.3732	.6366	.7366
8/15	1.875	1 7/8	1.675	.9375	1.2874	.5968	.6906
4/7	1.750	1 3/4	1.795	.8750	1.2016	.5570	.6445
8/13	1.625	1 5/8	1.933	.8125	1.1158	.5173	.5985
2/3	1.500	1 1/2	2.094	.7500	1.0299	.4775	.5525
16/23	1.437	1 7/16	2.185	.7187	.9870	.4576	.5294
8/11	1.375	1 3/8	2.285	.6875	.9441	.4377	.5064
16/21	1.312	1 5/16	2.393	.6562	.9012	.4178	.4834
4/5	1.250	1 1/4	2.513	.6250	.8583	.3979	.4604
16/19	1.187	1 3/16	2.646	.5937	.8156	.3780	.4374
8/9	1.125	1 1/8	2.792	.5625	.7724	.3581	.4143
16/17	1.062	1 1/16	2.957	.5312	.7295	.3382	.3913
1	1.000	1	3.142	.5000	.6866	.3183	.3683
1 1/15	.937	15/16	3.351	.4687	.6437	.2984	.3453
1 1/7	.875	7/8	3.590	.4375	.6007	.2785	.3223
1 3/13	.812	13/16	3.866	.4062	.5579	.2586	.2993
1 1/3	.750	3/4	4.189	.3750	.5150	.2387	.2762
1 5/11	.687	11/16	4.569	.3437	.4720	.2189	.2532
1 1/2	.666	2/3	4.712	.3333	.4577	.2122	.2455
1 3/5	.625	5/8	5.026	.3125	.4291	.1989	.2301
1 7/9	.562	9/16	5.585	.2812	.3862	.1790	.2071
2	.500	1/2	6.283	.2500	.3433	.1592	.1842
2 2/7	.437	7/16	7.181	.2187	.3003	.1393	.1611
2 1/2	.400	2/5	7.854	.2000	.2746	.1273	.1473
2 2/3	.375	3/8	8.377	.1875	.2575	.1194	.1381
3	.333	1/3	9.425	.1666	.2289	.1061	.1228
3 1/5	.312	5/16	10.053	.1562	.2146	.0995	.1151
3 1/2	.285	2/7	10.995	.1429	.1962	.0910	.1052
4	.250	1/4	12.566	.1250	.1716	.0796	.0921
4 1/2	.222	2/9	14.137	.1111	.1526	.0707	.0818
5	.200	1/5	15.708	.1000	.1373	.0637	.0737
5 1/3	.187	3/16	16.755	.0937	.1287	.0597	.0690
6	.166	1/6	18.850	.0833	.1144	.0531	.0614
7	.142	1/7	21.991	.0714	.0981	.0455	.0526
8	.125	1/8	25.133	.0625	.0858	.0398	.0460
9	.111	1/9	28.274	.0555	.0763	.0354	.0409
10	.100	1/10	31.416	.0500	.0687	.0318	.0368
16	.062	1/16	50.265	.0312	.0429	.0199	.0230

TABLE A, ANGLE OF TOP.

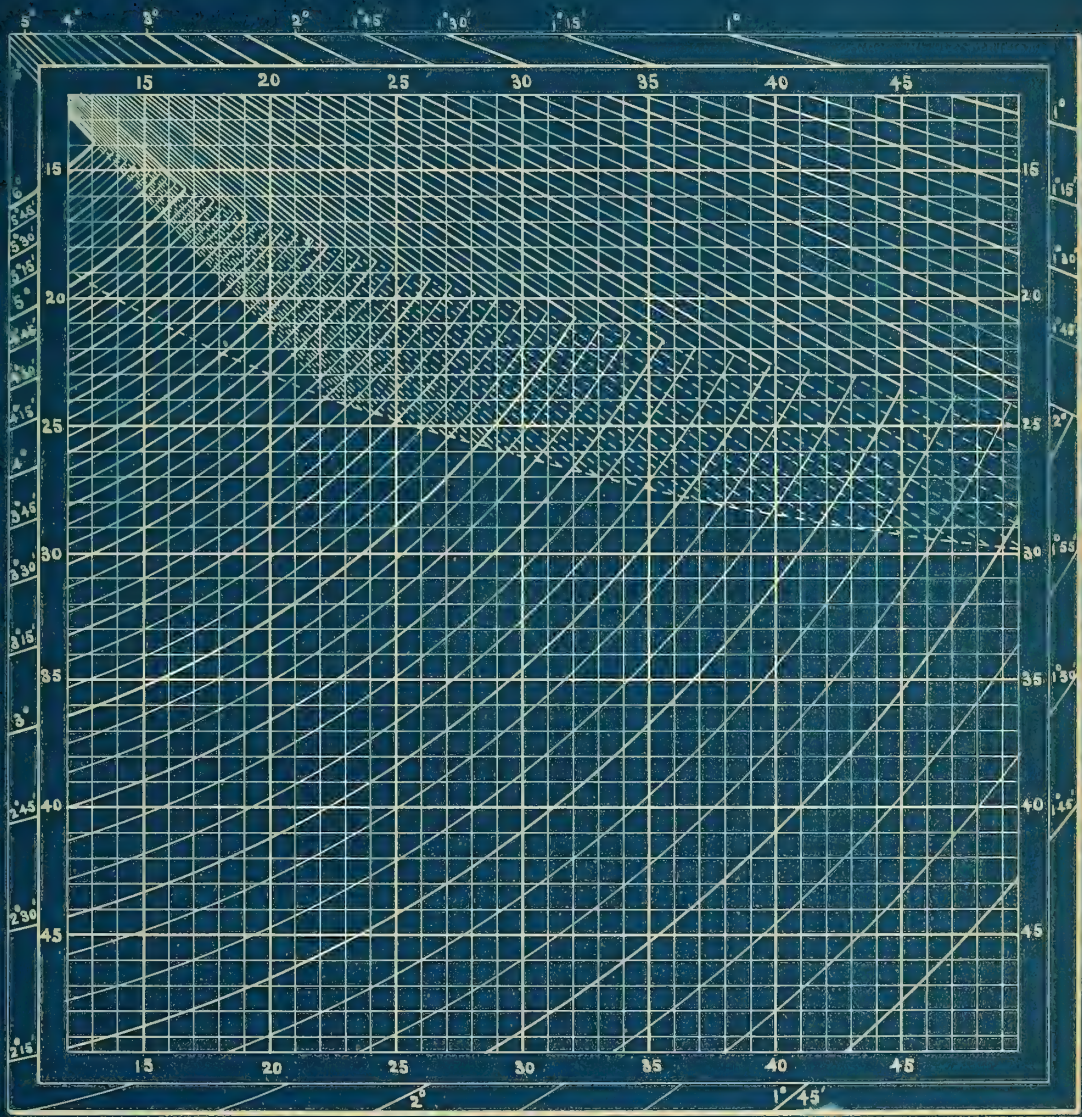


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TABLE A', ANGLE OF TOP

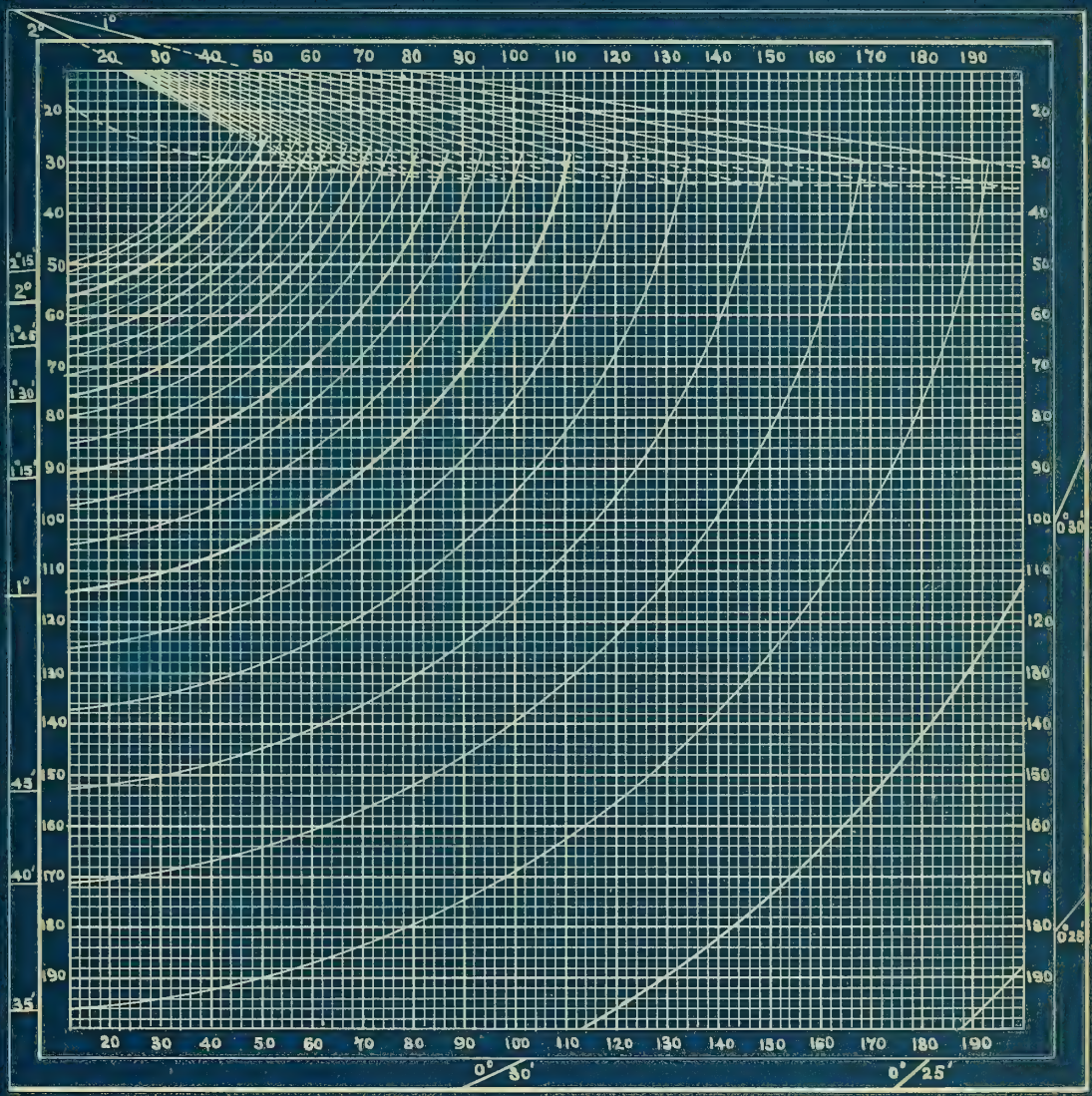

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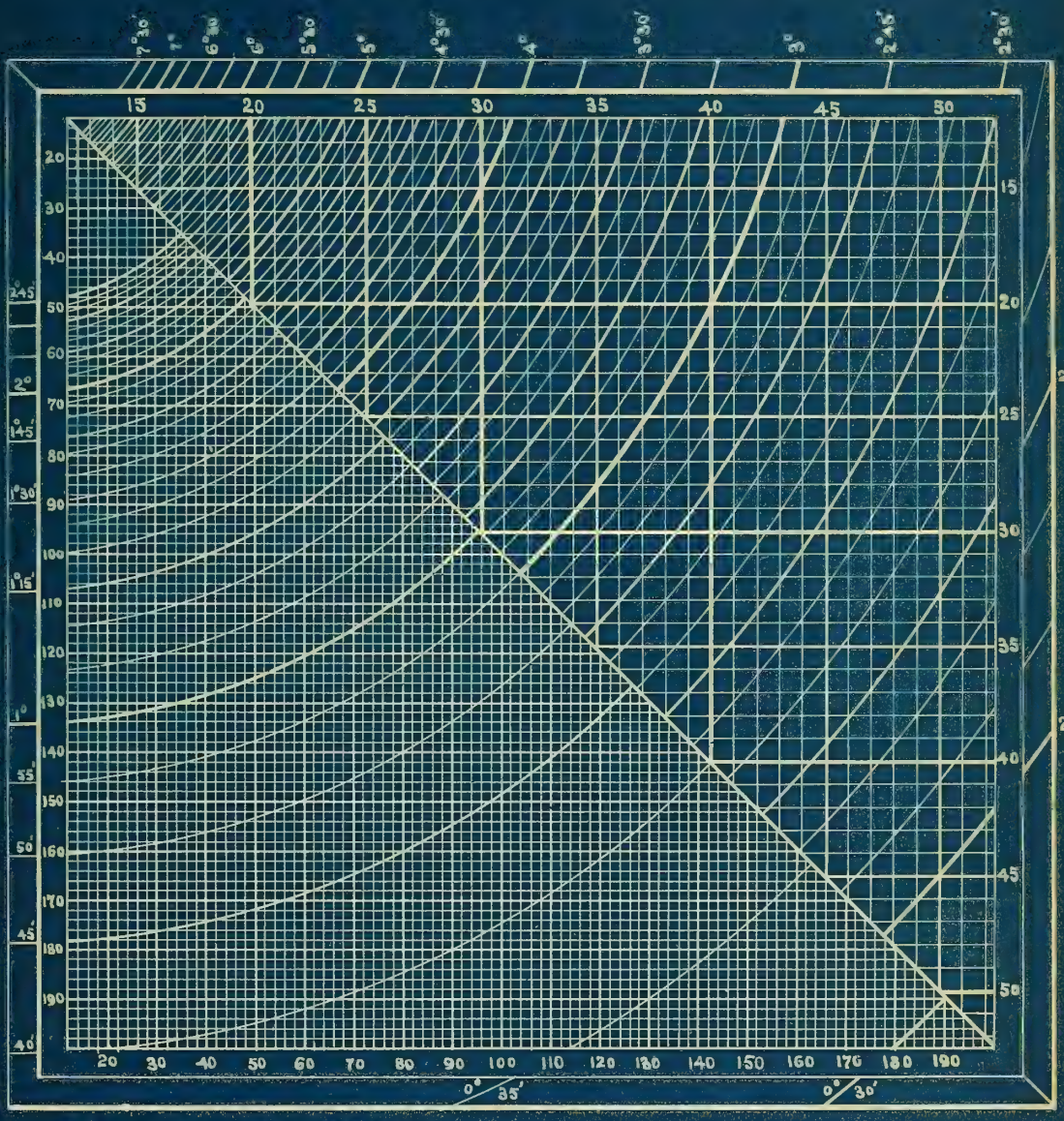
TABLE B, ANGLE OF BOTTOM



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Diameter - Increment,

2 DP.

Rule: The ratio being given or determined, to find the Outside-Diameter, divide figures given in table for large and small gear by Pitch (P) and add to Pitch-Diameter.

Ratio.		Gears: LARGE	Gears: SMALL	Ratio.		Gears: LARGE	Gears: SMALL	Ratio.		Gears: LARGE	Gears: SMALL
1.00	1:1	1.41	1.41	1.65		1.05	1.70	4.40		.45	1.94
1.05		1.37	1.42	1.67	5:3	1.03	1.72	4.50	9:2	.44	1.95
1.07		1.36	1.43	1.70		1.01	1.73	4.60		.42	1.95
1.10		1.35	1.44	1.75	7:4	.99	1.74	4.80		.41	1.96
1.11	10:9	1.34	1.46	1.80	9:5	.97	1.75	5.00	5:1	.39	1.96
1.12		1.33	.46	1.85		.95	1.76	5.20		.38	1.96
1.13	9:8	1.33	1.47	1.90		.93	1.77	5.40		.37	1.96
1.14	8:7	1.32	1.49	1.95		.91	1.78	5.60		.36	1.97
1.15		1.31	1.50	2.00	2:1	.89	1.79	5.80		.34	1.97
1.16		1.30	1.51	2.10		.87	1.80	6.00	6:1	.33	1.97
1.17	7:6	1.30	1.52	2.20		.84	1.81	6.20		.32	1.97
1.18		1.29	1.53	2.25	9:4	.82	1.82	6.40		.31	1.97
1.19		1.28	1.53	2.30		.80	1.83	6.60		.30	1.97
1.20	6:5	1.28	1.54	2.33	7:3	.78	1.84	6.80		.29	1.98
1.23		1.27	1.55	2.40		.76	1.85	7.00	7:1	.28	1.98
1.25	5:4	1.25	1.56	2.50	5:2	.75	1.86	7.20		.27	1.98
1.27		1.25	1.57	2.60		.73	1.86	7.40		.27	1.98
1.29	9:7	1.24	1.58	2.67	8:3	.71	1.87	7.60		.26	1.98
1.30		1.22	1.59	2.70		.69	1.87	7.80		.26	1.98
1.33	4:3	1.20	1.60	2.80		.67	1.88	8.00	8:1	.25	1.98
1.35		1.18	1.61	2.90		.65	1.89	8.20		.24	1.98
1.37		1.17	.61	3.00	3:1	.63	1.91	8.40		.24	1.98
1.40	7:5	1.16	1.62	3.20		.60	1.92	8.60		.23	1.98
1.43	10:7	1.15	1.63	3.33		.58	1.92	8.80		.23	1.98
1.45		1.13	1.65	3.40		.56	1.92	9.00	9:1	.22	1.99
1.50	3:2	1.11	1.66	3.50	7:2	.54	1.93	9.20		.22	1.99
1.53		1.10	1.67	3.60		.52	1.93	9.40		.21	1.99
1.55		1.09	1.67	3.80		.50	1.94	9.60		.21	2.00
1.58		1.08	1.68	4.00	4:1	.49	1.94	9.80		.20	2.00
1.60	8:5	1.07	1.68	4.20		.47	1.94	10.00	10:1	.20	2.00

COMPARATIVE SIZES OF GEAR TEETH.

Rule for finding Angle of Top β and Instructions regarding Undercut.

See Table

Table A & A' can be used for Bevel and Mitre - Gears only.

I.

To find the Angle, the number of teeth in Pinion and Gear must be known.

The upper horizontal and the left perpendicular line read from 12 - 50, whereas, having inserted the numbers of the example (24 + 31) find the intersection. It will be seen that two intersections are possible, owing to the fact that the number of teeth of either gear can be placed on the horizontal or perpendicular number-lines. Two intersections. — If they fall both below the dotted line a - b, we have no undercut but find the angle by determining the intersection with the lower horizontal or left perpendicular line, of a circle, laid through these two points, concentric with the circles of the table. The intersection will fall between two angles of 15' difference. The relative position of the point of intersection then gives the wanted angle (2° 09').

II.

Undercut.

By Undercut in gears is understood a special formation of the teeth, which may be explained by saying that the elements of the teeth below the Pitch line (in the pinion) are nearer the Center-line of the teeth, than those on or above the Pitch-line. Bevel gears with Undercut are useless for patterns as they will not draw out of the sand. Such gears can not be cut with cutters. Certain combinations of Gear and Pinion require Undercut as the table will show. In such cases the difficulty can be overcome by pushing the tooth of the gear having Undercut (Pinion) beyond the Pitch-line a certain distance (making β larger and consequently also s), and cutting the tooth short below the P.-Line making β' smaller and also s + f). This is literally nothing else than cutting away the "nose" of the tooth. If now the gear in mesh with the first one, is correspondingly corrected, it will be seen that the gears still mesh, the difficulty being removed.

For gears with undercut the tables of tooth parts are of no use, as s and s + f have to be calculated with reference to the corrected β + β' of each gear

II.

Correction for Stars with Undercut.

If in a given example (with 21 = 43) in locating the intersections of the Numberlines as located in I, we find that one of them falls above the lowest line a - b, we have a case of undercut. The change to be made in the table according to I consists in correcting the angles of top and bottom.

The amount of correction is found with the table as follows:

a.) For the intersection below a - b find angle as in case without undercut (2° 23')

b.) For the intersection above a - b, lay through the point a line convergent with the oblique lines of the table and read angle on the upper horizontal line (2° 05')

c.) The difference of the angles is the amount of correction (0° 18').

Rule: For Angle of Top B:

Add correct on small gear, Subtract f. large

For Angle of Bottom B:

Add correct on large gear, Subtract f. small

Note: Should one of the intersections fall between the lowest line a - b and the uppermost line c - d, the correction can be omitted and the angle found according to I. In such a case the undercut is

not sufficient enough to interfere in Tooth Bevel Gears. However the correction may be determined and judgement in the special case will lead the case. If above c-d, correction must be made.

V.

Angle of Bottom β'.

The angle of bottom is found in a similar manner as the angle of Top, from table B.

The large sectional part of the table is but an enlargement of the upper left hand corner of the lower table. In case of interest the correction is made according to Rule under part I.

VI.

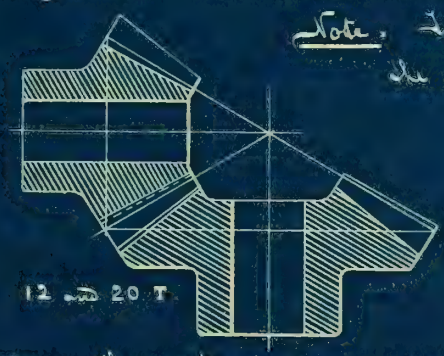
Undercut in Mitre-Gears.

It will be seen from the table, mitre-gears of 24 teeth and under, have undercut. It is very evident that in such a case the rules for corrections as given for gear & pinion can not be followed. It will further be seen from the table that only one reading can be obtained, which reading gives not the theoretical but the corrected angle of Top. To make use of this corrected angle in both mitres, would simply mean the shortening of the tooth above the Pitch-line, which

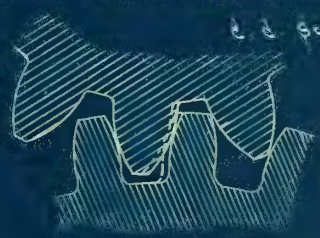
leaving it intact below the P. L. – But the part below the pitch-l. being the one subject to undercut, the difficulty still remains. It is therefore obvious that, to remove the undercut we have to correct (make smaller) the angle of bottom (β'), for the same amount β has been corrected. As the amount of correction can not be determined from the table we find it by calculating the theoretical angle of top by means of the formula:

$$\operatorname{tg} \beta = \frac{2 \sin \alpha}{N}$$

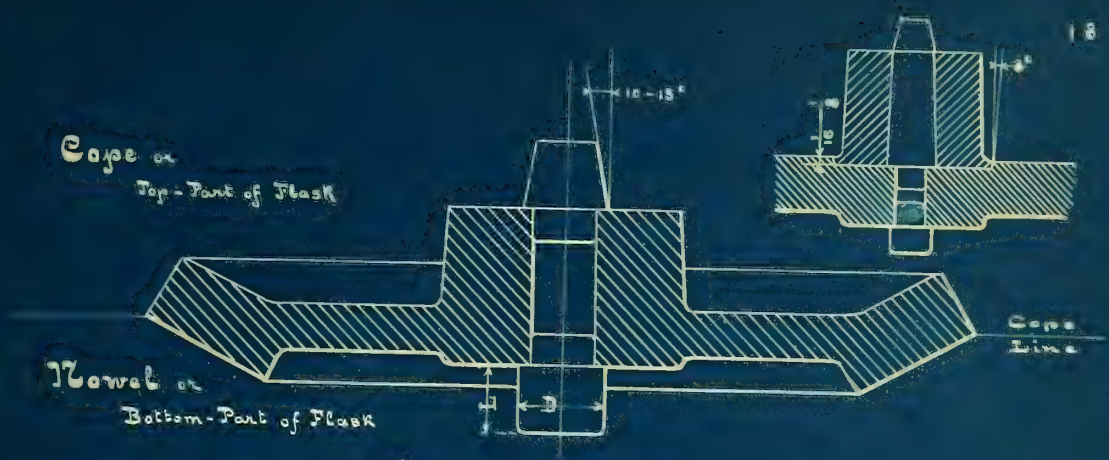
and compare the result with the angle found in the table. In now subtracting the correction from the angle of bottom as found in the respective table we obtain the corrected β' which determines the value of $s + f$. – It is therefore seen, that in order to do away with undercut in this case, we are compelled to shorten the teeth of both mitres in reducing s as well as $s + f$. – This is of course a disadvantage, which in pattern mitre-gears however can not be overcome. –



<u>Note.</u> In corrected gears the teeth of the pinion will sometimes be very



sharp, the workman can overcome this by leaving them thicker, and reducing 't' of the gear, as shown on the left. –



Gear Patterns.

To place in bevel-gears the best iron where it belongs the tooth-side of the pattern will always be in the nowel, no matter what shape the hubs. Hubs, if short, may be left solid on nowel; if long, should be made loose. A large hub should go on a long tapering arbor to prevent tipping in the sand. Give 1 deg. taper for draft on hubs when loose; 3 deg. when solid. Coreprints as a rule are made seperate, partly to allow the pattern to be turned on an arbor, partly for convenience, should it be desired to use different sizes. - Put Rap-out draw-holes as near to center as possible. — Make L = D for D from $\frac{3}{4}$ - $1\frac{1}{2}$" or even more, should hubs be very long. Otherwise if D more than $1\frac{1}{2}$", leave L = $1\frac{1}{2}$".

Iron patterns before using should be marked, rusted and waxed.

In patterns for cast gears of less than 8 in Pitch-Diameter, shrinkage is not taken into account.

Shrinkage for Cast-iron = $\frac{1}{8}$" per foot
for Brass = $\frac{3}{16}$ per foot.

Dimensions and form for Bevel Gear Cutters.

The data needed to determine the form and thickness of a bevel-gear-cutter are the following:

P = Pitch

N, n = Number of teeth for both gears

F = Length of face of tooth, measured on Pitch-L.

After having laid out a diagram of the pitch-cones a b c and a b f and laid off the width of face, the problem resolves itself into two parts:

Part I.

Determine proper Curve for Cutter.

It will be remembered, that in the involute system of cutters (the only one used for bevel-gears), a set of 8 different Cutters is made for each pitch, num-

being from # 1 to # 8 and cutting from a rack to 12 teeth. Each number represents the form of a cutter suitable to cut the indicated number of teeth. For instance # 4 Cutter (# 4 Curve) will cut 26 to 34 teeth. In order to find the curve to be used for gear and pinion we simply construct the normal Pitch-cones by erecting the perpendicular p q in b. We now measure the lines b q. + b p and taking them as radii, multiplying each by 2 and P we obtain a number of teeth which require for cutting certain curves, which in turn are the curves wanted for cutting our bevel-gear and pinion. From example we have

Gear: b q = $9\frac{3}{4}$" ; 2 × P × 9.75 = 97 T # 2 Curve

Pinion: b p = $3\frac{1}{2}$" ; 2 × P × 3.5 = 35 T # 3 "

Part II.

Determine thickness of Cutter.

It is very evident that a bevel-gear cutter can not be thicker than the width of the space at small end of tooth; the practice is to make cutter .005" thinner. Theoretically the cutting angle (b) is equal to Pitch-angle less angle of bottom (or b = α − β') Practically however better results are obtained by making b = α − β (substituting angle of tip for angle of bottom), and in calculating the depth at small end, to add the full clearance (f) to the obtained

working-depth, giving equal amount of clearance at large and small end. This is done to obtain a larger curve at bottom of tooth. — As the small end of tooth determines the thickness of cutter, we shall have to find the tooth-part-values at small end. From the diagram it will be seen that the values at large end are to those at small end as their respective Apex-distances (a b & a l). The numerical values of these can be taken from the diagram and the quotient of the larger in the smaller is the constant wherewith to multiply the tooth values at large end, to obtain those at small end. In our example we find:

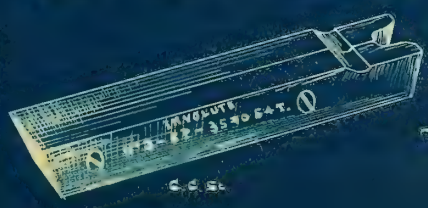
$$\frac{a\,l}{a\,b} = \frac{3.8}{5.8} = \underline{.655} = \text{Constant.}$$

For 5 P we have

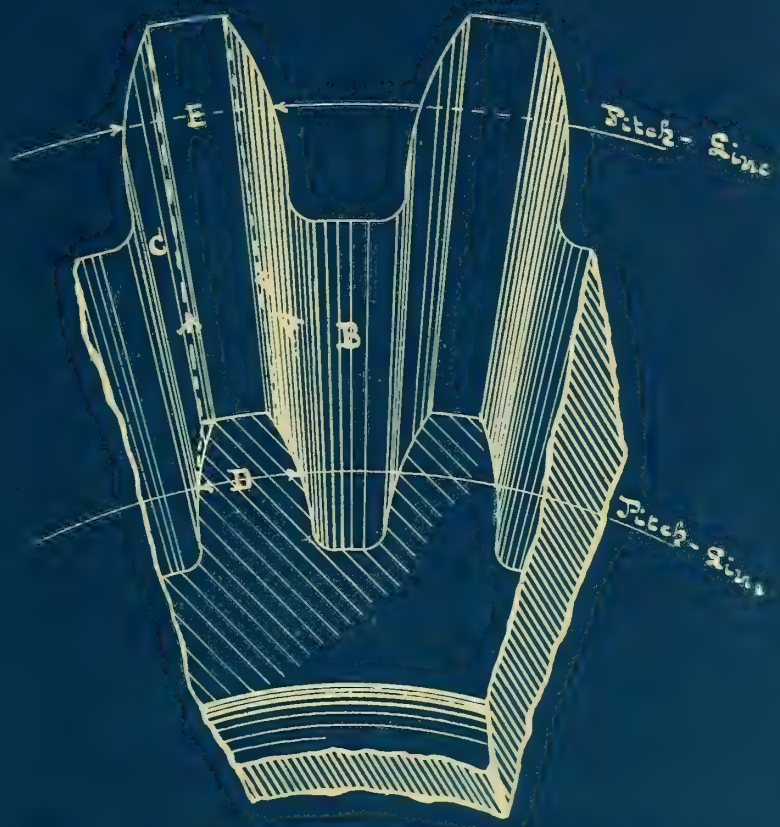
t	=	.3141	t'	=	.2057
s	=	.2000	s'	=	.1310
f	=	.0314	f	=	.0314
s+f	=	.2314	s'+f	=	.1624
D''+f	=	.4814	s'	=	.1310
			D'''+f	=	.2934

The workman now in making the cutter will use for Gear cutter-tool market 5 P # 2 and see that the thickness t' = .2057 at a depth s'+f = .1624 — . — From the foregoing it is evident that a Spur Cutter could not be used, since a bevel-gear cutter is thinner. —

No 1	cuts	135	– rock
2		55	– 134
3		35	– 54
4		26	– 34



No 5	cuts	21	– 25
6		17	– 20
7		14	– 16
8		12	– 13



Directions for cutting Bevel-Gears with Rotary Cutter.

In order to obtain good results, the gear-blanks must be of the right size and form. The following sizes of each cut of the teeth must be given the workman: D" + f, t" + s. He must also know the cutting-angle, and be provided with the proper tools

sufficient to measure teeth etc etc. — Most gear-cutting engines have vernier-scales or graduations by which the distance cutter are set off, each side from the center can be determined, as well as the distance the gear is revolved each side, after the central cut is made. If the machines are not provided with such scales, they can and should be attached.

The operations and adjustments of the machine are as follows:

1st. Set Cutter-head to proper cutting-angle.

2d. Place Cutter central with work.

3d. Set head- and work-spindles exactly on Zero-mark.

4th. As a matter of precaution make a scratch-mark for depth of cut on both, outside and inside edges of blank.

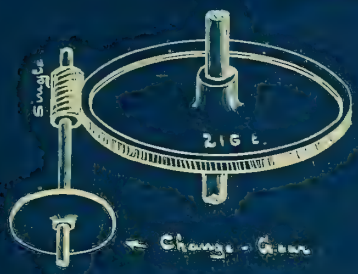
5th. Make two cuts to coincide with marks in depth.

6th. Set cutter off center an amount which to good judgement is not excessive (bearing in mind that an equal amount of stock is to be taken from each side of tooth), and make a cut on the side A, now index to the opposite side of tooth, set the cutter by vernier-scale an equal amount off on opposite side of center, and make a cut on side of tooth C. If the tooth is now thick on Pitch-line D, repeat the operation, till thickness is correct.

7th The tooth will now be thick at large end E. Proceed as follows: Set the cutter still more off center and revolve the gear in same direction in which the cutter is set off from Zero-line (which is indicated by Zero-mark on scale). Now make a cut, after which set cutter off center and rotate the gear an equal amount off center on opposite side (of center), index to opposite side of tooth C and make another cut. If the tooth is still thick at E, repeat the operation, bearing in mind, that the cutter must be moved and the gear rotated an equal distance on each side of their respective Zero-settings. ——

It is generally necessary to file faces above pitch-lines more or less, as indicated by broken lines F.F. This applies to pinions of less than 30 teeth. — For gears of coarser pitch than 4 diametral, it is best to make one cut around before attempting to obtain the tooth-thickness. —

The Indexing of any whole or fractional Number.



In indexing on a machine the question simply is: How many divisions of the Machine-index have to be advanced to advance a unit-division of the number required. To which is the answer = $\frac{\text{Divisions of Machine-Index}}{\text{Number to be indexed}}$

Suppose number of divisions in Index-wheel of machine to be 216.

Example I. Index 72

Answer $\frac{216}{72} = 3$ (3 turns of worm).

Example II. Index 15.708

$$\frac{216}{15.708} = \frac{216000}{15708} = 13 + \frac{11796}{15708}$$

If now we should put on Worm-shaft a change-gear having 15708 teeth, give the worm-shaft 13 turns out in addition thereto advance 11796 teeth of the change-gear (to give the fractional turn), we would have indexed correctly one unit of the given number and so solved the problem.

But practically a change-gear of 15708 teeth is an impossibility. We therefore reduce the fraction $\frac{11796}{15708}$ to lower approximate values by the process of continued fractions

$$\frac{11796}{15708} = \frac{5898}{7854} = \frac{2949}{3927} =$$

$$\frac{1}{1+\frac{1}{3+\frac{1}{65+\frac{1}{5}}}}$$

```
2949) 3927 ( 1
      2949
       978) 2949 ( 3
            2934
              15) 978 ( 65
                  975
                    3) 15 ( 5
```

$b \times c + a = d$
$b \times c' + a' = d'$

1	3	65	5	
$a = \frac{1}{1}$	$b = \frac{3}{4}$	$d = \frac{196}{261}$	$\frac{983}{1309}$	$= \frac{2949}{3927}$
$a' = 1$	$b' = 4$	$d' = 261$		

The fraction $\frac{196}{261}$ is a good approximation, and putting a change-gear of 261 teeth on worm-shaft we advance (besides the 13 full turns) 196 teeth to index our unit.

Supposing we have however only a gear of 242 teeth at our disposal, then

$$196 : 261 :: x : 242 \quad ; \quad x = 182$$

the fraction is obtained by advancing 182 teeth of our 242 tooth gear. —— (see Note).

Of course in using any but the correct fraction we commit an error every time we index a division. By the time we have indexed the circle we have multiplied the error by the number of divisions, which accumulated

error will show itself between the last and the first divisions. The best we can do in such a case is to as evenly distribute the errors in the circle as possible, which is done by advancing the change-gear one tooth more or less, at even intervals, which intervals may be determined.

Example III. Index 365.

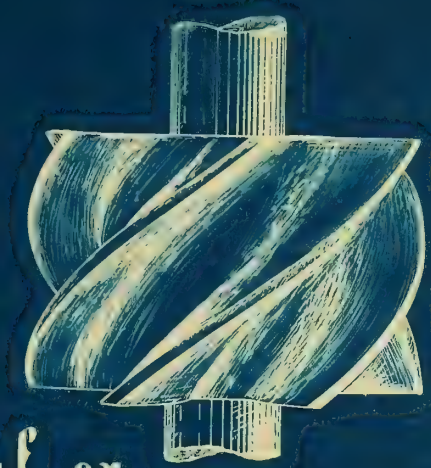
We simply treat the fraction $\frac{216}{365}$ as the similar one in Ex. II. There is a difference here from Ex. II, inasmuch as the fraction being a proper one, the worm-shaft turns but a fraction of a turn, to index a unit of the given number.

We may sometimes arrive at better results by trying a change-gear on hand. Suppose we possess gear 147. then $\frac{216}{365} = \frac{x}{147}$ where $x = 87 - \frac{3}{365}$, i.e. we advance 87 teeth of our change-gear and in indexing the complete circle we drop at even intervals 3 times one tooth of our change-gear.

Note. In considering the example

$196 : 261 :: x : 242$ we find $x = 182 - \frac{70}{261}$

i.e. in indexing 182 teeth of our change-gear at a time, we advance too fast, and will have indexed 70 teeth of our change-gear too many when completed the circle, we therefore have to drop at even intervals 70 times 1 tooth. –



Spiral or Screw = Gearing.

In spiral gearing the wheels have cylindrical pitch surfaces, but the teeth are not parallel to the axis. The line in which the pitch-surface intersects the face of a tooth is part of a screw-line or helix drawn on the pitch-surface. A screw wheel may have one or any number of teeth. A one toothed wheel corresponds to a one-threaded screw, a many-toothed wheel to a many threaded screw. The axis may be placed at any angle, but must be contained in two parallel planes, otherwise we obtain Skew-bevel-gears.

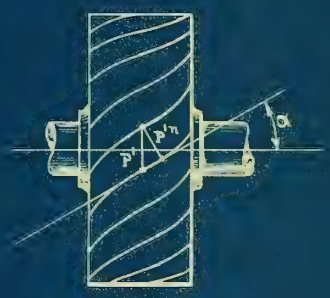
We shall consider Spiral-gears with:

I. Axis parallel
II. " at right angles
III. " at any angle.

Let there be:

N_a, N_b } Number of teeth in gears a b
C Center-distance
P Diametral Pitch
P' Circular Pitch
P^n Normal Diametral Pitch
P'^n Normal Circular Pitch
γ Angle of Axis
L_1 Exact Lead of Spiral on Pitchsurface
L_2 Approximate Lead of Sp. on Pitchsurface
T Number of teeth marked on Cutter to be used when teeth are to be cut on Milling Machine.

D Pitch-diameter
B Blank-diameter
α_a, α_b } Angle of teeth with Axis
t Thickness of teeth
s Addendum
$D''+f$ Whole Depth of tooth



Note: Letters a and b occuring at bottom of Notations, refer to Gears a and b.

1. Axis parallel.

Gears of this class are called twisted gears. The angle of teeth with axis in both gears must be equal, and the Spirals run in opposite hands. The angles are generally chosen small (seldom over 20°) to avoid excessive endthrust. Endthrust may however be entirely avoided by combining two pairs of wheels with right- and left-hand obliquity. Gears of this class are known as Herringbone-gears. They are noiseless running at high speed. —

II. Axis at right angles.

Here we must always have:

1) The teeth of same hand spiral and
2) The normal Pitches equal in both gears;
3) The sum of the angles of teeth with axis = 90°.

Choosing angle of teeth with axis.

(1.) If in a pair of gears the ratio of the number of teeth is equal to the ratio of the diameters, i.e. if the number of teeth in the two gears are to each other as the Pitch-diameters, then the angles of the spirals are 45° and 45°; for this condition fulfilled, the circular pitches of the two gears must be alike, which is only possible with angles 45°. In such a combination either gear may be the driver.

(2.) Is however the ratio of the diameters determined upon, and chosen larger or smaller than the ratio of the number of teeth, then the angles are:

$$\operatorname{tg} \alpha_a = \frac{D_a N_b}{D_b N_a} \qquad \operatorname{tg} \alpha_b = \frac{D_b N_a}{D_a N_b}$$

In such and following gears the velocity-ratio is measured by the number of teeth and not by the diameters.

(3.) Given N_a, N_b and C

If P'_a is made = P'_b, then we have case (1)

$$P' = \frac{C}{\frac{1}{2}(N_a + N_b)\pi}$$

But if P'_a is assumed, then:

$$P'_b = \frac{c\pi - \frac{1}{2} N_a P'_a}{\frac{1}{2} N_b}$$

and

$$tg\, \alpha_a = \frac{P'_a}{P'_b} \qquad tg\, \alpha_b = \frac{P'_b}{P'_a}$$

The gear whose P' or α is larger will be the driver, on account of the greater obliquity of the teeth.

(4.) Given N_a, N_b and c or D.

see case (7) under III, considering $\gamma = 90°$.

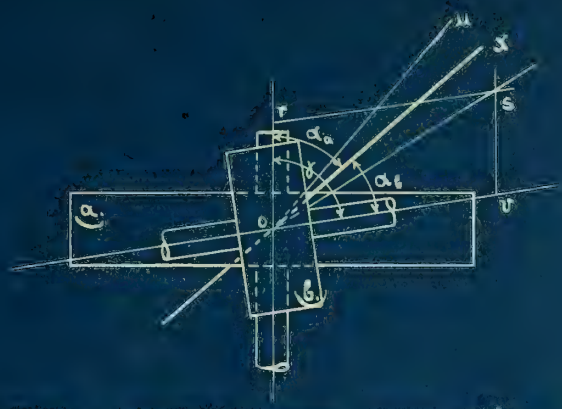
III. Axis at any angle. (γ)

(5.) Given case (1) under II, then Angles of Spirals $= \frac{1}{2}\gamma$, for the same reason.

(6.) Analogous cases for (2) and (3) under II may be worked out, when angles of axis $= \gamma$, they have been omitted, partly because the formulae are too cumbersome, partly because the cases are to some extent covered by (5) and (7).

(7.) Given N_a, N_b and c or one of the Pitch-diam.

We shall find the angles by a graphic method, which for all practical purposes is accurate enough. ro and vo are the axis of gears forming angle γ (see diagr. next page). On these axis we lay off lines ot and ov representing the ratio of the number of teeth (Velocity-ratio), so that $N_a : N_b :: rs : sv$



and construct parallelogram orsv. Then according to Mc Cord *) the angles formed by the tangent so in the Pitch-contact o with the axis of the gears insures the least amount of sliding. – In bi-secting angle γ by tangent uo and using angles produced in this manner we equally distribute the end-thrust on both shafts. Both methods have their advantages, to profit by both we select angles α_a and α_b, produced by tangent ox, bi-secting angle uos.

Thus we have when angles are found and c given

$$P'^n = \frac{2 c \pi (\cos \alpha_a + \cos \alpha_b)}{N_a + N_b}$$

and when D_a given

$$P'^n = \frac{D_a \pi \cos \alpha_a}{N_a} \quad \text{and}$$

$$D_b = \frac{P'^n N_b}{\pi \cos \alpha_b}$$

*) Kniematics Pg. 278.

General Formulae.

$$\gamma = \alpha_a + \alpha_b$$

$$P_a'^{n} = P_b'^{n}$$

$$D = \frac{P'N}{\pi} \quad \text{or} = \frac{P'^{n}N}{\pi \cos\alpha}$$

$$B = D + 2s \quad \text{or} = \quad D + \frac{2}{P^{n}}$$

$$P' = \frac{D\pi}{N} \quad \text{or} = \frac{P'^{n}}{\cos\alpha}$$

$$P'^{n} = P' \cos\alpha$$

$$P^{n} = \frac{P'^{n}N}{\pi} \quad [\text{Pitch of Cutter}]$$

$$s = \frac{P'^{n}}{\pi} \quad \text{or} = \frac{1}{P^{n}}$$

$$t = \frac{P'^{n}}{2}$$

$$D''+f = 2s + \frac{t}{10}$$

$$T = \frac{N}{\cos^2\alpha} \quad (\text{see Note 1.})$$

$$L_1 = \frac{NP'}{\operatorname{tg}\alpha} \quad \text{or} = \frac{N\pi}{P \operatorname{tg}\alpha} \quad \text{or} = \frac{NP'^{n}}{\operatorname{tg}\alpha \cos\alpha}$$

$$L_2 = \frac{10\, W G_2}{S\, G_1} \quad (\text{see Note 2.})$$

Note 1. Cutters of regular involute system.

Use						
№ 1 Cutter for T from	135	up.		№ 5 Cutter for T from	21	to 25
2	55	to	134	6	17	20
3	35		54	7	14	16
4	26		34	8	12	13

Note 2. Gears used on spiral-head and lent for Brown & Sharpe Milling-Machine:

W . Number of teeth in gear on Worm
G_1 . " " " " " " " 1st Stud
G_2 . " " " " " " " 2d " "
S . " " " " " " " Screw.

Should a spiral-head of different construction be used, the formula would not hold.

Internal Gearing.

A little reflection will show that a tooth of an internal or annular gear is the same as the space of a spur-external gear. — Internal gear and pinion of the epicycloidal form of tooth must differ by at least 15 teeth, and by 60 teeth if of the involute form. Gears fulfilling this condition will present no difficulties. Their Pitch-Diameter is found as in regular spur-gears and the inside-diameter is equal to Pitch-Diameter less 2 s.

If however the difference in the number of teeth between gear and pinion less than 15, say 6, or even 2 or 1, then we may construct the tooth-outline in 2 different ways, both based on the epicycloidal system, the involute failing us, except under special conditions. —

A. To better explain the first method let us suppose the case in Fig. 1, where the difference in gear and pinion is more than 15 teeth. Here the point O of the "Describing-circle" B (whose diam. in the best practice of the present day is equal to the pitch-radius of a 15 tooth-gear, of the same pitch as the gears in question) generates the cycloid O, O', O², O³ etc when rolling on pitch-circle L L of gear, forming the face of tooth; and when

Fig. 1.

6 P.
60 & 34 T.

rolling on the outside of L L the flank of the tooth. In like manner is the face and flank of the pinion-tooth produced, by B rolling outside and inside of E E (Pitch-circle of pinion). — A little study of the diagram (where the face and flank of a gear-tooth are produced) will show the describing-circle B divided into 12 equal parts and circles laid through these points (1, 2, 3 etc.) concentric with L L. We now lay off on L L the distances 0-1, 1-2, 2-3 etc of the circumference of B and obtain points 1', 2', 3' etc. [Ordinarily it is sufficient to use the cord.] It will now readily be seen that B in rolling on L L will successively come in contact with 1', 2', 3', etc, C meanwhile moving to C', C², C³ etc (Points on radii through 1', 2', 3', etc) and the generating point O advancing to O', O², O³ etc being the intersections of B (with C', C², C³ etc. as centers) and the circles laid through 1, 2, 3 etc. Points O, O', O², O³, etc connected with a curve give the face of the tooth; in like manner the flank is obtained.

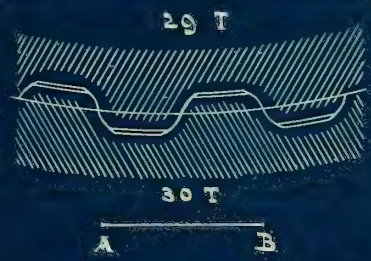
In precisely this manner the form of tooth is obtained, when the difference of teeth in gear and pinion is less than 15, with the exception that the diameter of describing-circle B is

$$= \frac{1}{2}\left(\frac{N - n}{P}\right)$$ where

P = diametral pitch, N and n number of teeth in gears.

The distances of the tooth above and below the pitch-line as well as the thickness t are determined as in regular spur-gears by the pitch, except when the difference in gear and pinion is very small, where we obtain a short tooth as in Fig. 2. In such a case the height of tooth is arbitrary and only conditioned by the curve. In internal gears it is best to allow more clearance at bottom of tooth than in ordinary spur-gears.

Fig. 2.



T' = 1"

In a construction of this kind it is suggested to draw the tooth-outline many times full size and reduce by photography. An equally multiplied line AB will help in reducing.

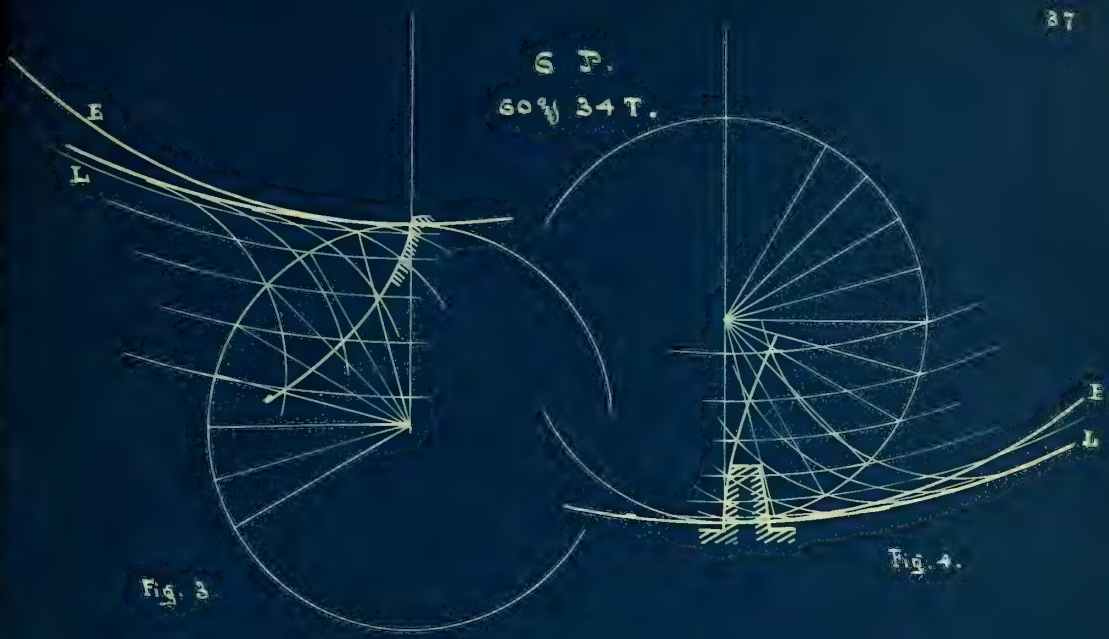


Fig. 3

Fig. 4.

B. The difference between gear and pinion being very small it is sometimes desirable to obtain a smooth action by avoiding what is termed the "friction of approching action"*) This is done, the pinion driving, by giving gear only flanks, Fig 3; and the gear driving, by giving gear only faces Fig 4. In both these cases we have but one describing circle, whose diameter is equal to the difference of the two pitch-diameters. The construction of the curve is precisely the same as described under A. The describing-circle has been divided into 24 parts simply for the sake of greater accuracy. —

6.

*) Mc Cord: Kinematics Pg. 107, 108.

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